Set Theory

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- Functions are relations (a set of ordered pairs, a subset of the Cartesian product of some two sets) that have a uniquely determined element in the second set (the range) for every single element in the first set (the domain). A function is identical when all elements of the triple are identical; identically, when the pairs corresponding the each domain value is identical.
- An inverse relation can only be a function when the original relation is a bijection as it must have a uniquely determined element (injective) in the domain that corresponds to every single element (surjective) in the range.
- Lets momentarily forget such conditions. In terms of composite functions, If the composite of two certain functions give birth to an identity function, the first of the two functions must be surjective as its image is squeeze-theoremed (haha) between the two range-sets (the smaller one is to be the image of the second function's range under the first function, as this is certainly to be within the domain of the first function, and the bigger one is to be the actual range). The second of the two functions must be injective, as two range values being identical necessarily results the two values of their preimage being identical.
- Injectivity can be seen as the alignment of the strings such that a single note can be played by a single key; surjectivity can be seen as all possible notes being able to be played.
- Set theory is all about sizes! When we made functions, it was all building up to the fact that it enables us to compare the size of two sets that it connects; an exceptionally powerful idea when it comes to infinite sets.
- An infinite set is a set where its part is as numerous as the whole.
- When proving a single element taken out of a infinite set is infinite, divide the cases into when the element is in and when it is out of the range of the function that enables the set to be infinite. When it is in it, a simple codomain and domain restriction is all that is needed. When it is out of it, connect the preimage to a arbitrary value outside the image of the function and the image of this newly defined function is properly constructed to be a proper subset of the stated set.
- When proving that a set is denumerable, you are trying to create a map that is injective that maps onto a single line or a single plane.
- Cardinal numbers are like suits and numbers on a card in a card deck. When you shuffle the numbers and spread them out, cardinal numbers exists for all numbers, explicit for zero and finite

numbers, and for infinite numbers, its more of a investigative process where we would rather think about them by comparing sizes.

- There are two variations for the definition of "A is smaller than B" for cardinal numbers. The first variation mentions that the size of A is equal to a subset of B and that B is not compareable in size to any portion of A (and therefore that A is smaller). The second variation is different in that instead of comparing the size of all portions of A, it only compares the size of B with the totality of A and says that they are not equal. The equivalence can be proven by contrapositive of the situation when the first condition is false.
- The Schröder-Bernstein theorem introduces us to conversations about injective functions (albeit the condition themselves are provided through bijections). It proves that if two sets are of the same size with subsets or one another, they themselves are the same size. When proving this theorem, it is necessary to prove a certain lemma; that if a set is injective to a subset of itself, there exists a certain bijection that connects the subset and the set.
- Corollary C is what actually explicitly uses the SB theorem to compare sizes through injective functions. It states that a injective function exists from A to B if and only if A is smaller or equal to B.
- Cantor's theorem proves that the set of all subsets of a certain set is always bigger than the set itself. It uses the definition of cardinal numbers and proves the second part of it, by creating a imaginary bijection from the original set to the power set. The set S is arbitrarily defined as the set of all elements that are not in its image. By self referencing and mentioning the inevitable existence of preimage of S, we create a contradiction where the image of this preimage is in itself, but defined to be not in itself. The Continuum hypothesis hypotheses that there is no cardinal number between the set of natural numbers and the power set of the natural numbers, which we find out to be equal to the cardinality of real numbers.

References

[1] S.Y.T. Lin and Y.F. Lin. *Set Theory: An Intuitive Approach.* Houghton Mifflin, 1973.