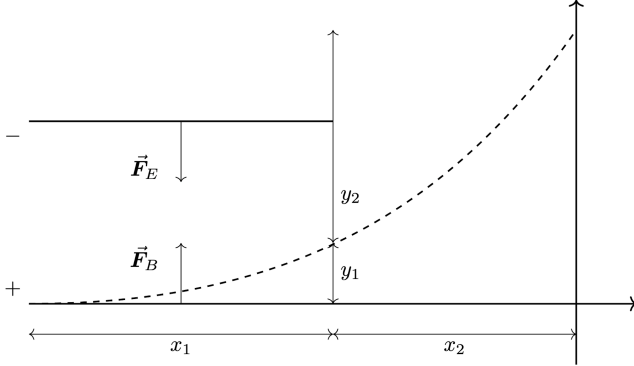


Modern Physics

Dane Jeon

I. QUANTIZATION OF CHARGE, LIGHT, AND ENERGY (11TH, APR)

1. *J.J. Thomson's experiment* proved the existence of the electron through measurements of charge per mass (q/m) with cathode rays.



The derivation for the vertical distance travelled ($y_1 + y_2$) is as follows.

$$\begin{aligned} y_1 + y_2 &= \frac{1}{2}at^2 + v_y t \\ &= \frac{1}{2} \left(\frac{eE}{m} \right) \left(\frac{x_1}{v_x} \right)^2 + \left(\frac{eE}{m} \right) \left(\frac{x_1}{v_x} \right) \left(\frac{x_2}{v_x} \right) \\ &= \left(\frac{e}{m} \right) \left(\frac{B^2}{E} \right) \left(\frac{1}{2}x_1^2 + x_1 x_2 \right) \end{aligned}$$

2. Millikan's Oil-drop experiment then allowed the measurement of the charge of a single electron, conclusively proving the *quantisation of electric charge* in units of $e = 1.6 \times 10^{-19}$ C. Another notable fact about Millikan is that he also failed to disprove Einstein's theory for the photoelectric effect (the quantisation of energy).
3. The quantum nature of radiation (light) was revealed from the study of black bodies by Max Planck. Black bodies first absorb radiation, increasing its constituent atoms' kinetic energy, rise in temperature, and radiate light to decrease its own temperature, reaching thermal equilibrium. Ideal black bodies absorb all of the incident radiation; *Stefan's empirical law*, discovered in 1879, gives the power radiated per unit area by the black body in terms of its temperature like the following.

$$R = \alpha T^4$$

Here, the constant α is known as Stefan's constant,

approximately $5.6703 \times 10^{-8} \text{ Wm}^{-2}\text{K}^4$. Stefan empirically derived this formula whereas Boltzmann separately derived it theoretically from classical thermodynamics, giving the law an alternate name: the Stefan-Boltzmann law.

4. *Wien's displacement law* describes the radiation emitted by a black body, stating that the energy density of the radiated light peaks at different wavelengths for different temperatures of the black body, with the explicit relationship described as follows.

$$\lambda_{\max} = \frac{b}{T}$$

The constant b has a value of $2.898 \times 10^{-3} \text{ mK}$.

5. The radiation power per unit area (in terms of the black body's energy density) is derivable as follows. We first try to find the energy contribution of infinitesimal volume element dV .

$$\begin{aligned} dU &= u dV \\ &= u (r^2 dr \sin \theta d\theta d\phi) \end{aligned}$$

Taking the black body as a cavity, the portion of the energy reaching the small hole of size dA during a time dt becomes

$$dU = u (r^2 c dt \sin \theta d\theta d\phi) \times \frac{dA \cos \theta}{4\pi r^2}$$

Dividing both sides by $dA dt$ gives the power of the radiation per unit area, R

$$\begin{aligned} dR &= u (r^2 c \sin \theta d\theta d\phi) \times \frac{\cos \theta}{4\pi r^2} \\ &= \frac{cu}{4\pi} \sin \theta \cos \theta d\theta d\phi \end{aligned}$$

$$\begin{aligned} R &= \frac{cu}{4\pi} \int_0^{\pi/2} \frac{1}{2} \sin 2\theta d\theta \int_0^{2\pi} d\phi \\ &= \frac{1}{4} cu \end{aligned}$$

The boundary values for the azimuth angle and the polar angle is carefully set based on the geometry of the integrated space (analogous to integrating a hemisphere with the origin being the center of the sphere).

II. QUANTIZATION OF CHARGE, LIGHT, AND ENERGY (16TH, APR)

1. The above conversation was on the energy per unit volume. We now turn our focus to spectral energy density, the energy per unit wavelength. The energy density distribution is given by the Rayleigh-Jeans equation, classically derived as the following. Considering the cubic cavity model from above with length L and considering the electromagnetic waves to be contained inside the cavity as standing waves, their possible one dimensional wavelengths become

$$L = \frac{1}{2}\lambda n$$

for any natural number n . Their consequent wave numbers becomes

$$k = \frac{2\pi}{\lambda} = \frac{\pi n}{L}$$

Considering the three dimensional case, the relationship between the angular frequency and the wave number (spacial frequency) is implemented as

$$\begin{aligned} \frac{\omega^2}{c^2} &= k_x^2 + k_y^2 + k_z^2 \\ &= \left(\frac{n_x\pi}{L}\right)^2 + \left(\frac{n_y\pi}{L}\right)^2 + \left(\frac{n_z\pi}{L}\right)^2 \end{aligned}$$

$$\frac{L^2\omega^2}{\pi^2c^2} = n_x^2 + n_y^2 + n_z^2$$

Assuming non-negativity of any of the numbers n_i and considering the two directions for polarisation, we multiply $2/8$ to the spherical "volume" of the possible modes (or total number of possible modes with the wave having a frequency less or equal to ω).

$$\frac{\pi}{3} \left(\frac{L\omega}{\pi c}\right)^3 = \frac{L^3\omega^3}{3\pi^2c^3}$$

The total number of possible modes per unit volume thus becomes the expression above divided by L^3 . Using the chain rule, we obtain the infinitesimal mode density for a small band of wavelengths from λ to $\lambda + d\lambda$.

$$\begin{aligned} \frac{dn}{d\lambda} d\lambda &= \frac{\omega^2}{\pi^2c^3} d\omega = \frac{\omega^2}{\pi^2c^3} \frac{2\pi c}{\lambda^2} d\lambda \\ &= \frac{4\pi^2c^2}{\lambda^2\pi^2c^3} \frac{2\pi c}{\lambda^2} d\lambda = \frac{8\pi}{\lambda^4} d\lambda \end{aligned}$$

Note that the size of the infinitesimal was used for exchanging $d\omega$ to $d\lambda$. The next step employs the equipartition principle, stating that each de-

gree of freedom of a vibrating particle has an energy of $k_B T$. The (spectral) energy density function in terms of lambda thus becomes

$$u(\lambda) = \frac{8\pi k_B T}{\lambda^4}$$

This theoretical derivation didn't align with empirical observations in low frequencies, and this misalignment was at the time named the ultraviolet catastrophe.

2. In attempts to reconcile, Plank derived *Plank's radiation law* assuming that (1) the oscillators in the cavity walls could not have a continuous distribution of possible energies but only discrete energies and that (2) the energy distribution function has an exponential decaying form by increasing energy. The average energy of a particle then becomes, instead of $k_B T$,

$$\bar{E} = \sum_{n=0}^{\infty} (n \times hf)(A \exp(-n \times hf/k_B T))$$

where the most right term represents the fraction of the oscillators that have the energy $n \times hf$. As the sum should equal 1, we can easily find A to be

$$1 - \exp(-hf/k_B T)$$

We then notice that the n dependent terms can be gathered to have the form of the derivative of $n \exp(-nx)$ with $x = hf/k_B T$.

$$\begin{aligned} \bar{E} &= (hf)(1 - \exp(-hf/k_B T)) \frac{\exp(-hf/k_B T)}{(1 - \exp(-hf/k_B T))^2} \\ &= \frac{hf \exp(-hf/k_B T)}{1 - \exp(-hf/k_B T)} \\ &= \frac{hc/\lambda}{\exp(-hc/\lambda k_B T) - 1} \end{aligned}$$

Thus, the energy density function becomes

$$\begin{aligned} u(\lambda) &= \frac{8\pi hc}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1} \\ &= \frac{8\pi hc}{\lambda^5} \exp(-hc/\lambda k_B T) \text{ as } \lambda \rightarrow 0 \\ &= \frac{8\pi hc}{\lambda^5} \frac{\lambda k_B T}{hc} = \frac{8\pi k_B T}{\lambda^4} \text{ as } \lambda \rightarrow \infty \end{aligned}$$

as required by the Rayleigh-Jeans equation and empirical observations. To summarise, we have 3 main laws that describe black-body radiation.

I. *Stefan-Boltzmann Law*

$$R = \sigma T^4$$

II. *Wein's Displacement Law*

$$\lambda_{\max} T$$

III. *Planck's Radiation Law*

$$u(\lambda) = \frac{8\pi hc}{\lambda^5} \frac{1}{\exp(hc/\lambda k_B T) - 1}$$

III. QUANTIZATION OF CHARGE, LIGHT, AND ENERGY (18TH, APR)

1. We refer back to the conversation of Einstein and the *photoelectric effect*. Einstein described the observations of Heinrich Hertz through suggesting that there exists a certain minimum energy that is required for the electrons to escape a photoelectric metal surface, and that small packets of light called quanta is responsible for the frequency dependency of electron emissions.

$$eV_0 = hf - \Phi$$

2. Further evidence of the correctness of the photon was furnished by Arthur H. Compton, measuring the scattering of X-rays by free electrons. X-rays, before this, was discovered by Wilhelm K. Roentgen in 1895 while working with cathode-ray tubes. After Max von Laue found that diffraction of X-rays were possible with crystals, W.H. Bragg (father) and W.L. Bragg (son) found the Bragg condition in 1912. The spectral distribution of X-rays were also a topic of interest for many physicists, having a particular characteristic spectrum depending on the target material.
3. The *Compton effect*, discovered in 1923 and awarded the Nobel prize in 1927, suggests the particle nature of photons. Compton's equation provides the change in wavelength of a photon as a function of the scattering angle θ . The derivation is as follows. Taking \mathbf{p}_1 and \mathbf{p}_2 as the momentum of the photon before and after the collision and \mathbf{p}_3 as the momentum of the stationary electron after the collision,

$$\mathbf{p}_1 = \mathbf{p}_2 + \mathbf{p}_3$$

$$\begin{aligned} p_3^2 &= (\mathbf{p}_2 - \mathbf{p}_1)^2 \\ &= \mathbf{p}_2^2 + \mathbf{p}_1^2 - 2\mathbf{p}_1 \cdot \mathbf{p}_2 \end{aligned}$$

In scalar form,

$$p_3^2 = \left(\frac{hf_1}{c}\right)^2 + \left(\frac{hf_2}{c}\right)^2 - 2\left(\frac{hf_1}{c}\right)\left(\frac{hf_2}{c}\right)\cos\theta$$

$$(p_3 c)^2 = (hf_1)^2 + (hf_2)^2 - 2(hf_1)(hf_2)\cos\theta$$

On the other hand, energy conservation suggests (subscript e abbreviated)

$$hf_1 + mc^2 = hf_2 + ((pc)^2 + (mc^2)^2)^{1/2}$$

$$(hf_1 - hf_2)^2 + (mc^2)^2 + 2(hf_1 - hf_2)(mc^2)$$

$$= (pc)^2 + (mc^2)^2$$

$$(pc)^2 = (hf_1 - hf_2)^2 + 2(hf_1 - hf_2)(mc^2)$$

Comparison implies

$$-2(hf_1)(hf_2)\cos\theta = -2(hf_1)(hf_2) + 2(hf_1 - hf_2)(mc^2)$$

$$mc^2(f_1 - f_2) = hf_1 f_2 (1 - \cos\theta)$$

$$\frac{1}{\lambda_2} - \frac{1}{\lambda_1} = \frac{h}{mc}(1 - \cos\theta)$$

h/mc is called the Compton wavelength, approximately 2.426×10^{-12} m. Note that the (1) photoelectric effect, the (2) Compton effect, and (3) pair production are three distinct ways that matter interact with light.

IV. THE NUCLEAR ATOM (30TH, APR)

1. J.J. Thomson, known for his Nobel prize for the discovery of electron (1897), created the Jellium (or the chocolate chip cookie) model. It was the initial popular model of the atom before Rutherford's. Afterwards, Rutherford's nuclear model (1911) was discovered. He was able to do this by shooting alpha particles at a malleable gold foil, and observing large deflections of the alpha particles only explainable by a concentrated charge on the center. The reason for why electrons were not pulled in was only explainable classically by centripetal motion, and the planetary model of the atom was born. This was later proved to be impossible theories in electromagnetism.
2. Bohr Postulated that (1) *electrons move in certain stable circular orbits without radiating dependent on quantized angular momentum* given as follows

$$L = mvr = \frac{nh}{2\pi} = n\hbar$$

and that (2) *atoms radiate when electrons make a tran-*

sition from one allowed orbit to another allowed orbit.

$$hf = E_i - E_f$$

The derivation for the possible energies of the hydrogen atom, which accurately predicted atomic spectra, goes as follows.

$$F = k \frac{Ze^2}{r} = \frac{mv^2}{r}$$

$$v = \left(\frac{kZe^2}{mr} \right)^{1/2}$$

$$L = mvr = n\hbar$$

(here, we use Bohr's first postulate)

$$r = \frac{n\hbar}{mv} = \frac{n\hbar}{m} \left(\frac{mr}{kZe^2} \right)^{1/2}$$

$$r^2 = \frac{n^2\hbar^2}{m^2} \frac{mr}{kZe^2}$$

$$r = \frac{n^2\hbar^2}{mkZe^2} = \frac{n^2}{Z} \left(\frac{\hbar^2}{mke^2} \right) = \frac{n^2}{Z} a_0$$

The constant $a_0 = 0.529\text{\AA} = 0.0529 \text{ nm}$ is known as the *Bohr radius*.

$$E = \frac{1}{2}mv^2 - \frac{kZe^2}{r} = -\frac{1}{2} \frac{kZe^2}{r}$$

$$E = -\frac{1}{2} \frac{kZe^2}{r} = -\frac{1}{2} kZe^2 \frac{mkZe^2}{n^2\hbar^2} = -\frac{mk^2Z^2e^4}{2n^2\hbar^2} = -E_0 \frac{Z^2}{n^2}$$

Like so, we derived the *allowed energies of electron orbits*. The constant

$$E_0 = \frac{1}{2} \frac{mk^2e^4}{2\hbar^2} = 13.6 \text{ eV}$$

is known as the *binding energy* or the *ionization energy*.

$$\frac{1}{\lambda} = \frac{hf}{hc} = \frac{E_0}{hc} Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) = Z^2 R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Where the constant $R = 1.097 \times 10^7 \text{ m}^{-1}$ is *Rydberg constant*. The accuracy of the prediction led Bohr to get his Nobel prize.

V. THE WAVELIKE PROPERTIES OF PARTICLES (2ND, MAY)

1. The *correspondence principle* states that: "in a region of large quantum numbers, classical calculations and quantum calculations must yield the same results". For large n in Bohr's model, the transition from n to $n-1$ yields a change of energy of:

$$\begin{aligned} \Delta E &= -E_0 Z^2 \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \\ &= -E_0 Z^2 \left(\frac{1}{n^2} - \frac{1}{(n-1)^2} \right) \\ &= -E_0 Z^2 \frac{n^2 - 2n + 1 - n^2}{n^2(n-1)^2} \\ &= -E_0 Z^2 \frac{-2n}{n^4} = E_0 Z^2 \frac{2}{n^3} \end{aligned}$$

2. The *fine structure constant* α can be expressed as

$$\alpha = \frac{ke^2}{\hbar c} = \frac{e^2}{4\pi\epsilon_0\hbar c} \sim \frac{1}{137}$$

The binding energy E_0 of above can therefore be expressed as

$$E_0 = \frac{mk^2e^4}{2\hbar^2} = \frac{mc^2k^2e^4}{2\hbar^2c^2} = \frac{1}{2}mc^2\alpha^2 = 13.6 \text{ eV}$$

Here, the rest mass mc^2 of the electron is taken as 0.511 MeV. We equate h/mc to be the Compton wavelength, the wavelength associated with a photon scattered by an individual particle.

$$a_0 = \frac{\hbar^2}{mke^2} = \frac{\hbar}{mc} \frac{\hbar c}{ke^2} = \frac{\hbar}{m\alpha} = \frac{1}{2\pi} \frac{1}{\alpha} \lambda_c$$

3. Louis de Broglie, in 1924, hypothesized that particles act like waves with a *frequency* $f = E/h$ and a *wavelength* $\lambda = h/p$ (the de Broglie wavelength). For photons, $E = pc$ and $E = hf$, which satisfies the hypothesis. The hypothesis doesn't satisfy for large masses, as the wavelength becomes nearly immeasurable. Back to the Bohr model of the electron (quantized angular momentum), $mvr = n\hbar$, and

$$2\pi r = \frac{2\pi n\hbar}{mv} = \frac{h}{p}$$

Thus, together with the de Broglie Hypothesis, we can see how the Bohr model can be interpreted as electrons creating standing waves in its orbit. The Davisson-Germer experiment then provided evidence to the hypothesis (Thompson's son, G.P. Thompson, and Davisson was together awarded a Nobel prize for their con-

tributions regarding electron diffraction). Another notable experiment on proving the wave-like properties of the electron was the double slit interference experiment, which proved the interference property of electrons. Like so, it was shown that *electrons and all other particles exhibit the wave properties of interference and diffraction.*

4. Waves are well known to be defined through the classical wave equation,

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2}$$

Here, assuming a sinusoidal shape (harmonic), generally,

$$\begin{aligned} y(x, t) &= y_0 \cos(kx - \omega t) \\ &= y_0 \cos 2\pi \left(\frac{x}{\lambda} - \frac{t}{T} \right) \\ &= y_0 \cos \frac{2\pi}{\lambda} (x - vt) \end{aligned}$$

Here, the angular frequency is defined as $\omega = 2\pi f = 2\pi/T$, the wave number is defined as $k = 2\pi/\lambda$, and *wave* or *phase velocity* is defined as

$$v_p = \lambda f = \frac{\omega}{k}$$

The phase velocity is the speed at which information travels through the wave. A pulse can be obtained through a superposition of a group of such waves, which we call wave packets. Adding two harmonic waves as an example,

$$\begin{aligned} y(x, t) &= y_0 \cos(k_1 x - \omega_1 t) + y_0 \cos(k_2 x - \omega_2 t) \\ &= 2y_0 \cos \left(\frac{(k_1 x - \omega_1 t) - (k_2 x - \omega_2 t)}{2} \right) \\ &\quad \times \cos \left(\frac{(k_1 x - \omega_1 t) + (k_2 x - \omega_2 t)}{2} \right) \\ &= 2y_0 \cos \left(\frac{\Delta k}{2} x - \frac{\Delta \omega}{2} t \right) \\ &\quad \times \cos \left(\frac{k_1 + k_2}{2} x - \frac{\omega_1 + \omega_2}{2} t \right) \\ &= 2y_0 \cos \frac{\Delta k}{2} \left(x - \frac{\Delta \omega}{\Delta k} t \right) \cos (\bar{k}x - \bar{\omega}t) \end{aligned}$$

With the first term determining the envelope of the produced term within space, we come to the conclusion that the value $\Delta\omega/\Delta k$ is the wave packet's speed

(group velocity). Infinitesimally, this expression becomes $\partial\omega/\partial k$.

$$v_g = \frac{\partial\omega}{\partial k}$$

This group velocity can be seen as the particle's speed, implying that all matter can be seen as waves (matter waves). The phase velocities of the individual harmonic waves, on the other hand, are given by $\bar{\omega}/\bar{k}$. An approximation for $\Delta k \Delta x$ can be done by considering two adjacent root values for the equation above. As $\cos \Delta kx/2 = 0$, $\Delta kx/2 = \pi/2$, and $\Delta kx = \pi$. Taking $\cos \Delta kx'/2 = 0$, $\Delta kx'/2 = 3\pi/2$, and $\Delta kx' = 3\pi$. The value $\Delta k \Delta x$ thus approximates to 2π , and for a general wave packet, $\Delta x \Delta k \sim 1$. As $\Delta\omega/\Delta k = v_p = \Delta x/\Delta t$, we can also conclude that $\Delta\omega \Delta t \sim 1$. In this way, Heisenberg's uncertainty principle can also be seen as a statement about the wave-like property of small particles. Written separately, the *uncertainty relations* are

$$\Delta k \Delta x \sim 1$$

$$\Delta \omega \Delta t \sim 1$$

VI. THE WAVELIKE PROPERTIES OF PARTICLES (7TH, MAY)

1. For general wave packets, the quantities $\Delta x \Delta k$ and $\Delta \omega \Delta t$ are approximately 1, shedding light into Heisenberg's uncertainty principle. We now turn to specific forms of wave packets. An equation for a wave can be generally expressed through a Fourier cosine series, namely,

$$y(x, t) = \sum_i A_i \cos(k_i x - \omega_i t)$$

and for $t = 0$,

$$y(x, 0) = \sum_i A_i \cos k_i x$$

Extending from the argument above, the Fourier transform can be defined through the equations

$$f(x) = \int_{-\infty}^{\infty} F(k) e^{2\pi i k x} dk$$

$$F(x) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i k x} dx$$

A Gaussian wave packet, which we momentarily concentrate on, has the form of a normal distribution. This lets us see the standard deviation for position and wave number as precise definitions for "uncertainty". Starting with the proportionality between the displacement space function and an exponential function, we

can use the Fourier transform to also observe that the wave number space wave function is also a Gaussian distribution.

$$f(x) \propto \exp(-x^2/\sigma_x^2)$$

$$F(k) \propto \exp(-k^2/\sigma_k^2)$$

For the Gaussian wave packet, we state without explanation the product $\sigma_k\sigma_x$ is always exactly equal to $1/2$, giving the minimum possible uncertainty.

2. The *probabilistic interpretation* of the wave function states that The magnitude square of the wave function is proportional to the probability of observing a particle in the region x and $x + dx$.

$$P(x) dx = |\Psi(x)|^2 dx$$

3. Note that for light,

$$E = hf = h\omega/2\pi = \hbar\omega$$

$$p = h/\lambda = \hbar k$$

Armed with the uncertainty relations derived above and the analysis of Gaussian distributions, we arrive at the uncertainty principle that makes a statement about the "minimum" of such products. Using the equations above,

$$\Delta k \Delta x \sim 1 \rightarrow \Delta x \Delta p \sim \hbar$$

$$\Delta \omega \Delta t \sim 1 \rightarrow \Delta E \Delta t \sim \hbar$$

Defining Δk and Δx as standard deviations, the minimum is $\hbar/2$ and

$$\Delta x \Delta p \geq \frac{1}{2} \hbar$$

$$\Delta E \Delta t \geq \frac{1}{2} \hbar$$

which together is the *Heisenberg uncertainty principle*.

4. The one-dimensional particle in a box thought experiment successfully shows some consequences of the formulation above. Take $\Delta x = L$ and $\Delta p \geq \hbar/2\Delta x = \hbar/2L$. The definition of standard deviation suggests

$$\begin{aligned} (\Delta p)^2 &= (p - \bar{p})_{\text{av}}^2 \\ &= (p^2 - 2p\bar{p} + \bar{p}^2)_{\text{av}} \\ &= \bar{p}^2 - 2\bar{p}\bar{p} + \bar{p}^2 \\ &= \bar{p}^2 - \bar{p}^2 \end{aligned}$$

As the box is symmetric, $\bar{p} = 0$ as there will be as many particles moving left as to the right. We then obtain

$$(\Delta p)^2 = \bar{p}^2 \geq \frac{\hbar^2}{(2L)^2}$$

due to Heisenberg's uncertainty principle. The average kinetic energy can also be found to be

$$\bar{E} = \left(\frac{p^2}{2m} \right)_{\text{av}} = \frac{\bar{p}^2}{2m} \geq \frac{1}{2m} \frac{\hbar^2}{4L^2} = \frac{\hbar^2}{8mL^2}$$

The overall argument suggests that the *minimum energy of a particle in any kind of box cannot be zero*. This minimum energy is often called the zero point energy.

4. The total energy of a non-relativistic particle of mass m is

$$E = \frac{p^2}{2m} + V(x, t)$$

Taking p and E to be operators, $p \rightarrow \hbar\partial/i\partial x$ and $E \rightarrow -\hbar\partial/i\partial t$, we arrive at the Schrödinger equation,

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \Psi(x, t) + V(x, t) \Psi(x, t) = i\hbar \frac{\partial}{\partial t} \Psi(x, t)$$

We now check the equation's validity for a free particle (a harmonic wave function) where $V = V_0$ (constant). We take the trial solution

$$\Psi(x, t) = A \exp\{i(kx - \omega t)\}$$

This gives

$$\frac{\partial^2 \Psi}{\partial x^2} = -k^2 A \exp\{i(kx - \omega t)\}$$

$$\frac{\partial \Psi}{\partial t} = -i\omega A \exp\{i(kx - \omega t)\}$$

Substituting,

$$\frac{\hbar^2 k^2}{2m} A \exp\{i(kx - \omega t)\} + V_0 \exp\{i(kx - \omega t)\}$$

$$= \hbar\omega A \exp\{i(kx - \omega t)\}$$

$$\frac{\hbar^2 k^2}{2m} + V_0 = \hbar\omega$$

The probability, on the other hand, is

$$\begin{aligned} P(x, t) dx &= |\Psi(x, t)|^2 dx \\ &= \Psi^* \Psi dx \end{aligned}$$

With $\Psi(x, t)$ taken as the probability amplitude,

$$\int_{-\infty}^{+\infty} |\Psi|^2 dx = \int_{-\infty}^{+\infty} A^* A dx = 1$$

as

$$|\Psi|^2 = A^* \exp\{-i(kx - \omega t)\} \times A \exp\{i(kx - \omega t)\} = A^* A$$

We now employ separation of variables. Suppose that the potential function is purely a function of x ($V(x, t) = V(x)$), and

$$\Psi(x, t) = \psi(x)\phi(t)$$

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} [\psi(x)\phi(t)] + V(x)\psi(x)\phi(t) = i\hbar \frac{\partial}{\partial t} [\psi(x)\phi(t)]$$

$$-\frac{\hbar^2}{2m} \phi(t) \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi(x)\phi(t) = i\hbar \psi(x) \frac{\partial \phi}{\partial t}$$

$$-\frac{\hbar^2}{2m} \frac{1}{\psi(x)} \frac{\partial^2 \psi}{\partial x^2} + V(x) = i\hbar \frac{1}{\phi(t)} \frac{\partial \phi}{\partial t}$$

In the last line, the left hand side is purely a function of x whereas the right hand side is purely a function of t . Equating each differential equation to a separation constant, we can find a solution for $\phi(t)$.

VII. THE SCHRÖDINGER EQUATION (9TH, MAY)

- Note that $\phi(t) = \exp(-iEt/\hbar)$ is the time dependent part of the wave function. This is due to the fact that the frequency, according to de Broglie's relation, is given by E/h (solving the differential equation is redacted). Substituting this solution and expressing the Schrodinger equation in terms of operator notation,

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$\underbrace{\left[-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} + V(x) \right]}_{\text{operator}} \underbrace{\psi(x)}_{\text{eigenfunction}} = \underbrace{E}_{\text{eigenvalues}} \psi(x)$$

the above equation is called the *time independent Schrodinger equation* (as we necessarily assumed that the potential function was given as a constant through time), with the operator, eigenfunction, and eigenvalues as the above. Remember that the *normalisation condition* is,

$$\int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = 1$$

which is also expressible in terms of the $\psi(x)$ component as

$$\begin{aligned} |\Psi(x, t)|^2 &= \Psi^*(x, t)\Psi(x, t) \\ &= [\psi(x)\phi(t)]^* \phi(x)\psi(x)\phi(t) \\ &= \phi^*(t)\psi^*(x)\psi(x)\phi(t) \\ &= \phi^*(t)\phi(t)\psi^*(x)\psi(x) \\ &= |\psi(x)|^2 \end{aligned}$$

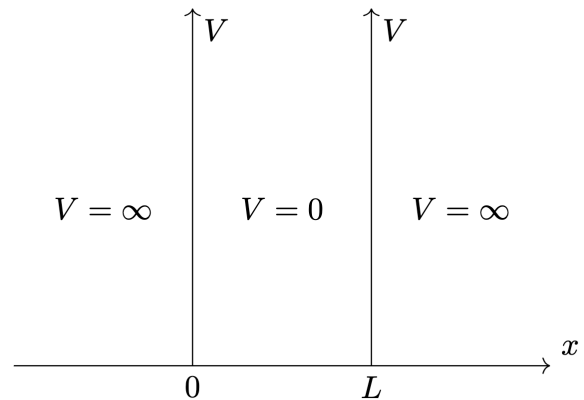
Some information used is that $\phi^*(t) = \exp(+iEt/\hbar)$ and $\phi(t) = \exp(-iEt/\hbar)$. For future reference, we note the *acceptability conditions* for $\psi(x)$:

- $\psi(x)$ must exist and satisfy the Schrödinger equation.
 - $\psi(x)$ and $d\psi/dt$ must be continuous.
 - $\psi(x)$ and $d\psi/dt$ must be finite.
 - $\psi(x)$ and $d\psi/dt$ must be single valued.
 - as $x \rightarrow \pm\infty$, it must be that $\psi(x) \rightarrow 0$ fast enough such that the normalisation integral is bounded.
- The *infinite square well* is a good example that illustrates the time-independent Schrodinger equation. Take

$$V = \begin{cases} 0 & 0 < x < L \\ \infty & x < 0 \text{ or } L < x \end{cases}$$

The time-independent Schrödinger equation becomes

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$



For $x < 0$ and $L < x$, the potential is $V(x) = \infty$,

forcing $\psi(x) = 0$. For the other case,

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi(x)$$

and thus,

$$\frac{d^2\psi}{dx^2} = -\frac{2mE}{\hbar^2}\psi = -k^2\psi$$

where $k^2 = 2mE/\hbar^2$ (note that the negative sign is place such that the solution is expressible as a symmetric trigonometric function). Two independent solutions therefore are expressible either as exponentials or sinusoidal functions,

$$\psi(x) = C_{1,2} \exp\{\pm ikx\} \quad \text{or} \quad C_1 \sin kx \quad \text{and} \quad C_2 \cos kx$$

We take the sinusoidal version for convenience, and the time independent solution becomes

$$\psi(x) = A \sin kx + B \cos kx$$

Enforcing the boundary conditions, the continuity of $\psi(x)$ at $x = 0$ and $x = L$ implies that at $x = 0$, $\psi(0^-) = 0 = \psi(0^+) = B$, giving $B = 0$. What about at L ? At $x = L$, $A \sin kL = \psi(L^-) = \psi(L^+) = 0$, and $\sin kL = 0$. We note that A must not be 0 due to probabilistic reasons, and therefore that $kL = n\pi$ for integers n . With index notation, $k_n = n\pi/L$, and as $k^2 = 2mE/\hbar^2$, we realise that energy levels differ for different solutions and that

$$k_n^2 = \frac{2mE_n}{\hbar^2} = \left(\frac{n\pi}{L}\right)^2$$

The different *allowed energy levels* become

$$E_n = n^2 \frac{\pi^2 \hbar^2}{2mL^2}$$

and the solution thus becomes

$$\psi_n(x) = A \sin\left(\frac{n\pi}{L}x\right)$$

for $0 < x < L$ and

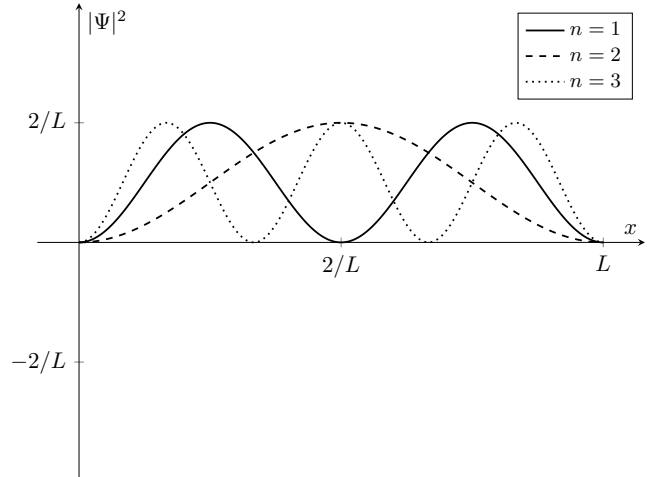
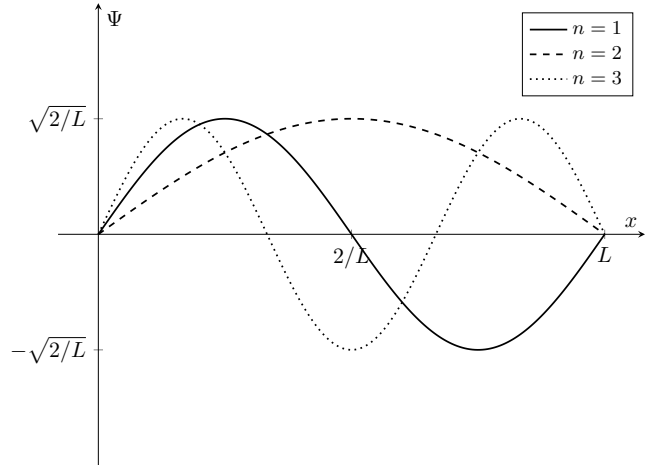
$$\psi_n(x) = 0$$

for $x < 0$ or $x > L$. We then perform integration, giving

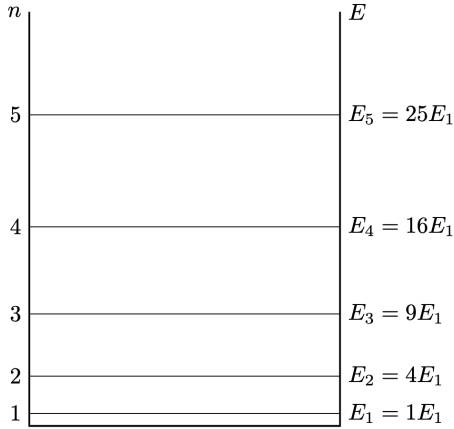
$$\begin{aligned} 1 &= \int_0^L |A \sin k_n x|^2 dx \\ &= A^2 \int_0^L \sin^2 k_n x dx \\ &= A^2 \int_0^L \frac{1 - \cos 2k_n x}{2} dx \\ &= A^2 \left(\frac{L}{2} - \frac{1}{2 \times 2k_n} \sin 2k_n x \Big|_0^L \right) \\ &= A^2 \left(\frac{L}{2} \right) \end{aligned}$$

We obtain $A = \sqrt{2/L}$, and therefore

$$\psi_n(x) = \begin{cases} \sqrt{2/L} \sin \frac{n\pi x}{L} & 0 < x < L \\ 0 & x < 0 \quad \text{or} \quad L < x \end{cases}$$



Taking $E_1 = \pi^2 \hbar^2 / 2mL^2$, the energy levels for the different n values become like the following.



Back to the de Broglie relation,

$$\lambda = \frac{h}{p} \quad \text{and} \quad p = \frac{h}{\lambda}$$

The condition for the standing waves in the well becomes

$$L = \frac{\lambda_n}{2} n$$

and

$$\lambda_n = \frac{2L}{n} \rightarrow P_n = \frac{nh}{2L} = \frac{n\hbar 2\pi}{2L} = \frac{n\pi\hbar}{L}$$

The possible energy levels are then

$$E_n = \frac{p_n^2}{2m} = \frac{1}{2m} \frac{n^2 \pi^2 \hbar^2}{L^2} = \frac{\hbar^2 \pi^2}{2mL^2} n^2$$

Which is of course identical to the result we obtained through the argument above! Remember that the complete wave function includes the time part,

$$\phi(t) = \exp\{-iE_n t / \hbar\}$$

and the solution becomes (take $E_n = \hbar \omega_n$)

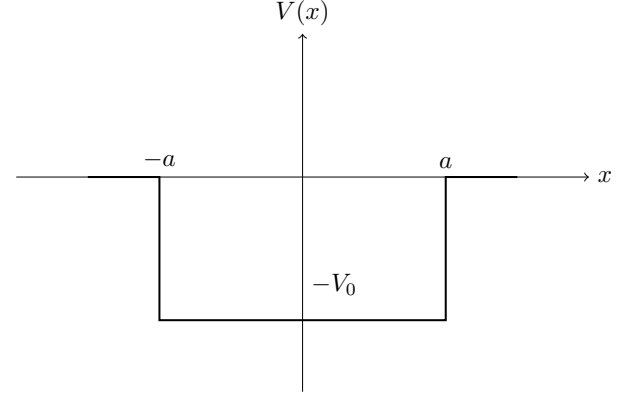
$$\begin{aligned} \Psi(x, t) &= \sqrt{\frac{2}{L}} \sin k_n x \exp\{-i\omega_n t\} \\ &= \sqrt{\frac{2}{L}} \frac{\exp\{ik_n x\} - \exp\{-ik_n x\}}{2i} \exp\{-i\omega_n t\} \\ &= \frac{1}{2i} \sqrt{\frac{2}{L}} [\exp\{i(k_n x - \omega_n t)\} - \exp\{-i(k_n x + \omega_n t)\}] \end{aligned}$$

This time part should never be forgotten.

3. The *finite quantum well*, a slightly more general case,

is constructed like the following. $V(x) = 0$ for $|x| > a$ and $-V_0$ for $|x| < a$.

$$V = \begin{cases} -V_0 & |x| < a \\ 0 & a < |x| \end{cases}$$



The Schrödinger equation for $|x| < a$ (inside the well),

$$-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) - V_0 \psi(x) = E \psi(x)$$

and

$$\frac{d^2}{dx^2} \psi(x) = -\frac{2m(V_0 + E)}{\hbar^2} \psi(x)$$

Taking the ansatz $\psi = \exp\{\pm ikx\}$, we obtain $k'^2 = 2m(E + V_0)/\hbar^2$. For the case where $|x| > a$ (outside the well),

$$\frac{\hbar^2}{2m} \frac{d^2}{dx^2} \psi(x) = |E| \psi(x)$$

Taking the ansatz $\psi = \exp\{k'x\}$ and we obtain $k'^2 = 2m|E|/\hbar^2$. $\psi(\pm\infty) \rightarrow 0$ implies $\psi(x) = \exp\{-\kappa x\}$ for $x > a$ and $\exp\{+\kappa x\}$ for $x < -a$, and by looking at the continuity of ψ and $d\psi/dx$ at $\pm a$, we arrive at the conclusion that

$$\psi(x) = \begin{cases} A \exp\{kx\} & x < -a \\ B \sin k'x + C \cos k'x & -a < x < a \\ D \exp\{-kx\} & x > a \end{cases}$$

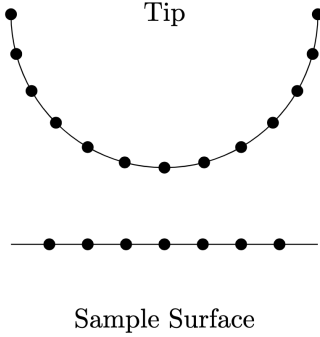
With this solution, we consider two cases. For the first case, we place a particle of kinetic energy $-V_0 < 0 < E$ in the well. When the particle is sent across the boundaries, few particles cross the boundary, but some get reflected. This situation can be seen identical to light being shot at a stack of three media, with different refractive indices ($n = 1.0, 1.5, 1.0$).

For the second case, we place a particle with a kinetic energy of $-V_0 < E < 0$ in the well. It is observable that there exists an amplitude beyond a , a phenomena

called quantum tunnelling.

VIII. THE SCHRÖDINGER EQUATION (14TH, MAY)

- The scanning tunneling microscope (STM) in particular uses a extremely sharp conducting tip that creates a potential between the tip and sample, inducing tunnelling of the electrons on a surface to the tip, allowing manipulation of electrons in the atomic scale.



- For the simple harmonic oscillator,

$$V(x) = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2x^2$$

with $\omega = \sqrt{k/m}$ and the *allowed energies* being

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kx_{\max}^2$$

with the minimum energy being 0. Here, the maximum x_{\max} is continuous for classical mechanics, but in the quantum realm, it is discrete, with the total energy as

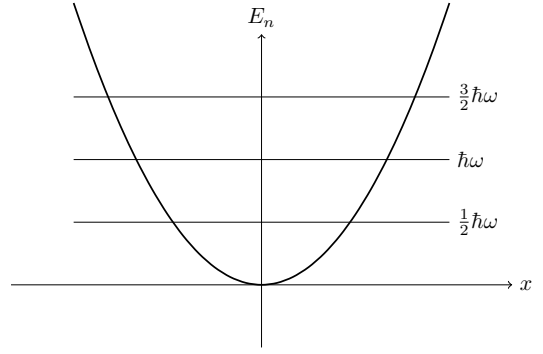
$$E_n = \left(n + \frac{1}{2}\right) \hbar\omega$$

with n being the natural numbers 1, 2, 3, ... Note that the energy levels are (1) quantized, (2) equally spaced, and (3) the minimum is given as $E_{\min} = \hbar\omega/2$ (zero-point energy) due to Heisenberg's uncertainty principle.

- In atomic physics, the hydrogen atom follows the time independent Schrödinger equation in 3 dimensions like the following.

$$-\frac{\hbar^2}{2m} \left[\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \right] + V(x, y, z)\psi = E\psi$$

Treating the hydrogen atom as a single particle, the



electrostatic attraction's potential becomes

$$V(r) = -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} = -\frac{Zke^2}{r}$$

the right hand side is due to the fact that the potential is given by the hydrogen atom. In spherical coordinates, the Schrödinger equation in rectangular coordinates become

$$-\frac{\hbar^2}{2m} \left[\frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial \psi}{\partial r} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial \psi}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi}{\partial \phi^2} \right] + V(r)\psi = E\psi \quad (*)$$

Using separation of variables,

$$\psi(r, \theta, \phi) = R(r)f(\theta)g(\phi)$$

On the test! When the solution is substituted into (*), we obtain

$$-\frac{\hbar^2}{2m} \left[fg \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + Rg \frac{1}{r^2 \sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{df}{d\theta} \right) + Rf \frac{1}{r^2 \sin^2 \theta} \frac{d^2 g}{d\phi^2} \right] + VRfg = ERfg$$

After rearrangement,

$$\left[\frac{1}{R} \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \frac{1}{f} \frac{1}{r^2 \sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{df}{d\theta} \right) + \frac{1}{g} \frac{1}{r^2 \sin^2 \theta} \frac{d^2 g}{d\phi^2} \right] = -\frac{2m}{\hbar^2} (E - V)$$

and

$$\frac{1}{R} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \frac{2mr^2}{\hbar^2} (E - V) = -\frac{1}{f} \frac{1}{\sin \theta} \frac{d}{d\theta} \left(\sin \theta \frac{df}{d\theta} \right) - \frac{1}{g} \frac{1}{\sin^2 \theta} \frac{d^2 g}{d\phi^2} \quad (**)$$

Which is exactly the form we required for r : a separable solution that is reduced to an ordinary differential equation. With foresight, we state that both sides equal $l(l+1)$, and

$$\frac{1}{g} \frac{d^2 g}{d\phi^2} = -l(l+1) \sin^2 \theta - \frac{\sin \theta}{f} \frac{d}{d\theta} \left(\sin \theta \frac{df}{d\theta} \right)$$

Again with a bit of foresight, we state that both sides are $-m^2$. The ϕ -component solution thus becomes

$$g_m(\phi) = \exp\{im\phi\}$$

where m is an integer. The single valued condition of Ψ implies $g(\phi + 2\pi) = g(\phi)$, thus requiring an integer. The θ -component solution

$$f(\theta) = f_{l,m}(\theta)$$

has the form of associated Legendre polynomials (the subset where $m = 0$ are the familiar Legendre polynomials), and the ϕ times θ -component,

$$Y_{l,m}(\theta, \phi) = f_{l,m}(\theta)g_m(\phi)$$

have solutions of the form of spherical harmonics. The radial component $R_{n,l}(r)$ have solutions of the form of associated Laguerre polynomials.

4. The angular momentum is particularly a good quantum number, and taking momentum as an operator ($p_x \rightarrow \hbar d/i dx$), we can obtain the magnitude of the angular momentum

$$|\mathbf{L}| = \hbar \sqrt{l(l+1)}$$

with l being in \mathbb{N}_0 .

IX. ATOMIC PHYSICS (16TH, MAY)

1. As discussed last time, the angular momentum is

$$\mathbf{L} = \mathbf{r} \times \mathbf{p}$$

The z -component angular momentum becomes an operator

$$L_z = \frac{\hbar}{i} \frac{\partial}{\partial \phi}$$

We must not forget the radial part, where $R \rightarrow R_{n,l}$ with the constants l and m being $l = 0, 1, 2, \dots$ $m = -l, -l+1, \dots, +l-1, +l$, with the m 's called the magnetic quantum numbers.

$$E_n = -\frac{1}{2} \mu c^2 (Z\alpha)^2 \frac{1}{n^2}$$

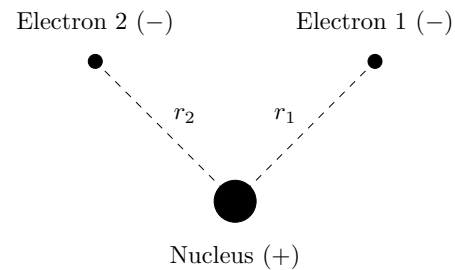
Lastly, the constant $n = 1, 2, \dots$ and are called principal quantum numbers. The quantum numbers can be

organised as

$$\begin{cases} n : \text{principle} & n = 1, 2, 3, \dots \\ l : \text{orbit} & l = 0, 1, \dots, n-1 \\ m : \text{magnetic} & m = -l, -l+1, \dots, +l \end{cases}$$

- The electron spin, $S^2 \rightarrow s(s+1)\hbar^2$ and $s = 1/2$. Also, $S_z = \pm\hbar/2$.
- There are Fermions and Bosons. For Fermions, $S = \text{odd}/2$, and examples are electrons, protons, and neutrons. For Bosons, $S = \text{integers}$, and examples are photons and the Higgs particle. Fermions follow the Pauli Exclusion Principle.
- The energy of the Helium atom is calculated as

$$E = \frac{P_1^2}{2m_e} + \frac{P_2^2}{2m_e} - \frac{kZe^2}{r_1} - \frac{kze^2}{r_2} + \frac{ke^2}{r_{12}}$$



The left contribution (the first term and the third term) accounts for $E_n = -13.6 \text{ eV} Z^2/n^2$ and the right contribution (the second term and the fourth term) accounts for $E_n = -13.6 \text{ eV} Z^2/n^2$. With $Z = 2$, $E_1 = 2 \times 2^2 \times -13.6 \text{ eV} + (\text{correction}) = -108.8 \text{ eV} + (\text{correction})$. Experimentally, $E_1 = -78.975 \text{ eV}$, which is a substantially large error. For Lithium ($Z = 3$), $n = 1$ states are full with 2 electrons, and the remaining 1 electron is on $n = 2$. These are called atomic shells, with $n = 1, 2, l = 0, 1, 2, 3, \dots, n-1$, and afterwards, there are s, p, d , and f shells. For the third electron,

$$V_{\text{eff}} = -\frac{kZ_{\text{eff}}e^2}{r}$$

and $Z_{\text{eff}} \sim 1.3$. This relative effect is called screening. We now turn our heads to carbon, with $Z = 6$. It is constructed as $1s^2 2s^2 2p^2$. The reason why carbon is so special is because carbon is divalent. $1s^2 2s^1 2p^3$ has a slightly higher energy, but making 4 bonds is more favourable. $5p^3$ hybridization is the $2s^1 2p^3$ acting like something it is not.

5. Now, we look at molecules, first at the most simplest case of H_2^+ .

$$V = -\frac{ke^2}{r_1} - \frac{ke^2}{r_2} + \frac{ke^2}{R}$$

X. SPECIAL LECTURE (TWO-DIMENSIONAL LAYERED MATERIALS) (21TH, MAY)

1. Some examples of low-dimensional systems are thin films (quantum wells) (2D), quantum wires (1D), quantum dots (nano-particles) (0D). What are ultimate 2D materials? Graphene is a perfect example (a single layer of carbon). Carbon nano-tubes and buckyballs all have unit elements made out of graphene. Historically, the existence of graphene was first postulated theoretically. Graphene has an interesting band structure. For a generally particle,

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} = \frac{\hbar^2}{2m}k^2$$

For a relativistic particle,

$$E = \sqrt{m^2c^4 + p^2c^2} = c\sqrt{m^2c^2 + \hbar^2k^2}$$

Through band theory, it can be seen how for graphene, the conduction bands and valence bands meet, creating a mass-less Dirac fermion, something only theoretically producible

$$E(\mathbf{k}) = \hbar v_F |\mathbf{l}|$$

Klein tunneling is an example of what graphene has been able to experimentally prove. The half-integer quantum Hall effect been also been able to proved through Graphene.

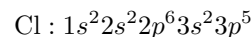
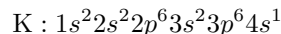
2. There are many advantages of graphene. It is (1) thin with single particle layers, (2) it can create electrons that act as if they have no mass, (3) it has very high electron speeds with 100 times of silicon, (4) it has high thermal conductivity, (5) it has the highest current density in high temperatures being 100 times the density for copper, (6) it is the strongest material being 100 times steel, (7) it has 98% of a light penetration rate, and (8) it is the most dense net where even Helium can't penetrate it. These are some reasons why Graphene is often called the "dream advanced material". Andre Geim and Konstantin Novoselov earned their Nobel prize for the discovery of Graphene in 2010.
3. There were many attempts in finding Graphene. The one that worked in 2004 was using Scotch tape. The way that this was observed was through interference of light due to the thin films of silicon below the graphene sample.
4. Chemical vapor deposition (CVD) is a way of creating graphene in area quantities. The roll-to-roll process was used to make a large area of graphene, capable of technologically making large touch screens. EU Graphene Flagship was a project that started in 2013 in the UK.
5. Graphene is important in the field of physics also, as it might replace silicon as transistors (as transistors increase, thermal energy dissipation increases), it might

become transparent electrodes for displays and touch-screens (indium tin oxides are currently used which have many limits such as not being flexible, rare, and weak), it might become electrodes for lithium batteries, or it might replace copper wires (there rare other examples, such as heat-ware, tennis rackets, coating for corrosion).

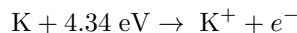
6. There are many different 2D materials, where MoS₂ is another 2D material worth noting. Currently, 2D layered materials is one field that is studied substantially. Recently, it has been discovered that Graphene acts as a superconductor when layered with each other in an angle. 2D magnetic material has also been recently studied.

XI. SOLID STATE PHYSICS (23RD, MAY)

1. For molecules, bonds are created such that it minimizes the energy. Two types of bonds are ionic bonds (for example salt KCl) and covalent bonds. K has the atomic number $Z = 19$ and Cl has the atomic number $Z = 17$. Their respective structures are



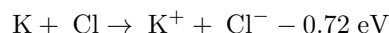
The energy that is required to take away a single electron is called the ionization energy.



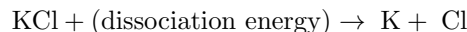
The energy released in combining a single electron is called the electron affinity.



Adding the two equations together,



However, we must also take into account the Coulomb interaction between K^+ and Cl^- . This creates a net attraction to create the ionic bond KCl. The energy required to take apart two atoms is called the dissociation energy.



2. Fluorine crystals (C⁶⁰) are put together through Van der Waals interactions. Amorphous solids are solids that are not crystalline (crystal in structure). There are also liquid crystals and quasicrystals, polymers, and simple crystal structures. There are many examples of simple crystal structures: the sodium chloride structure (also called the rock salt structure); the cesium chloride structure; the Hexagonal close-packed (hcp) struc-

ture, where LED's take their form as; and the diamond structure.

3. Lattices are defined as a collection of points that have translational symmetry. In this way, a crystal structure can be seen as a Bravais lattice with a certain basis, thus having periodic potential.

XII. SOLID STATES PHYSICS (28TH, MAY)

1. Single particle picture contains a free particle ($V = 0$) with the wave function

$$\psi = \exp\{ikx\}$$

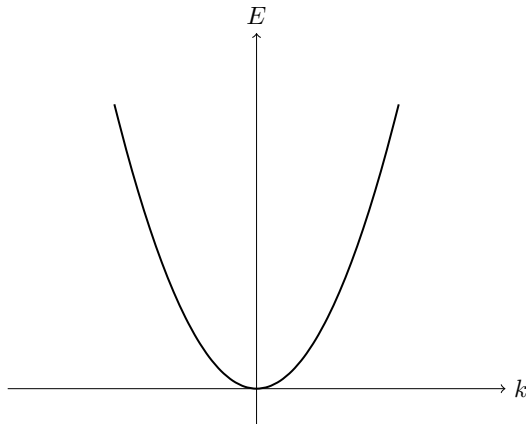
the Schrödinger equation becoming

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi$$

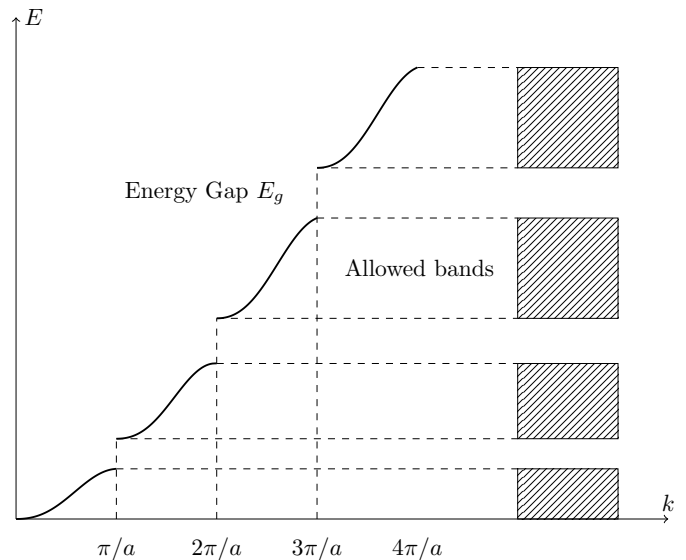
with the solved energy being

$$E = \frac{\hbar^2 k^2}{2m}$$

The energy function becomes (implying that all energies larger than 0 is possible)



2. In the case that the particle isn't free ($V \neq 0$), the energy function becomes like the following, and forbidden energy gaps and allowed energy gaps are now observable.
3. Bands can be classified for metals, insulators, and semiconductors. *Band theory* is used to distinguish one from another, where the individual energy levels are split into bands of allowed energies. In a conductor, the uppermost band containing electrons is only partially full, so there are many available states for excited electrons. In an insulator, the uppermost band containing electrons, the valence band, is completely full and there is a large energy gap between it and the next allowed band, the conduction band. In a semiconductor, the energy gap between the filled valence band and the empty



conduction band is small, so at ordinary temperatures an appreciable number of electrons are thermally excited into the conduction band.

4. Semiconductors, metals, magnetic materials, dielectric and ferroelectric materials, and superconductors are materials that are within the scope of interest of solid state physics.
5. Nuclides are nuclei that are not a part of an atom, but nuclei that exist independently. Their constituents are nucleons (protons and neutrons) and the numbers of nucleons, protons, neutrons are named the mass number, the atomic number (proton number), and the neutron number respectively. The variation of electrons lead to isotopes, while Radionuclides are materials whose nucleus are unstable. Examples of isotopes of hydrogen are deuterium in heavy water and tritium.
6. The line of stability shows nuclei that are stable. Nuclear radii is measured in femtometers, and the nucleus is not a solid object with a well defined surface, which is why we use effective radii.
7. The binding energy of the electron is given as

$$B_{\text{atomic}} = M_{\text{nucleus}}c^2 + Zm_e c^2 - M_{\text{atom}}c^2$$

for the nucleus,

$$\begin{aligned} B_{\text{nucleus}} &= Zm_p c^2 + Nm_n c^2 - M_{\text{nucleus}}c^2 \\ &= Zm_p c^2 + (Zm_e c^2 - B_{\text{atomic}}) + Nm_n c^2 \\ &\quad - M_{\text{nucleus}}c^2 - (Zm_e c^2 - B_{\text{atomic}}) \\ &\sim ZM_{\text{hydrogen}}c^2 + Nm_n c^2 - M_{\text{atom}}c^2 \end{aligned}$$

The binding energy per nucleon becomes B_{nuclear}/A and according to Yukawa's theory, it is what what

binds the nucleus is the strong force.

XIII. NUCLEAR PHYSICS (30TH, MAY)

1. The binding energy per nucleon graph has a maximum at iron.
2. Radioactivity was first discovered by Antoine Henri Becquerel in 1896, and the Nobel prize was awarded for this discovery along with P. and M. Curie.
3. There are different types of radioactive rays (α , ${}^4\text{He}$, β (e), γ (photon) rays). Radioactive decay is a statistical process, and therefore is impossible to predict.
4. The decay rate is given as

$$-\frac{dN}{dt} = \lambda N$$

$$N(t) = N_0 \exp\{-\lambda t\}$$

$$R = -\frac{dN}{dt} = \lambda N_0 \exp\{-\lambda t\} = R_0 \exp\{-\lambda t\}$$

the mean life is given as $\tau = 1/\lambda$ where $R(\tau) = R_0/e$ and $N(\tau) = N_0/e$. The half life can be calculated as

$$\frac{1}{2}N_0 = N_0 \exp\{-\lambda t_{1/2}\}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \tau \ln 2 = \frac{0.693}{\lambda} = 0.693\tau$$

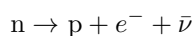
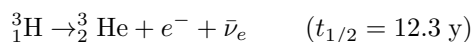
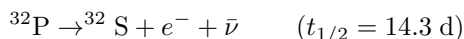
5. Alpha decay (for example ${}^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^4\text{He}$) contains a disintegration energy Q as

$$\Delta M = M_{\text{Th}} + M_{\text{He}} - M_{\text{U}}$$

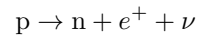
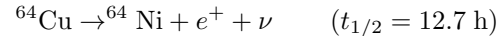
$$Q = -\Delta M c^2 = 4.25 \text{ MeV}$$

The half life of ${}^{238}\text{U}$ is 4.5×10^9 y and the half life of ${}^{228}\text{U}$ becomes 9.1 min. The amount of grams of uranium needed for reactions to occur every second. On the test!

6. In the case of beta decay, examples of the beta minus decays are



examples of beta plus decay are

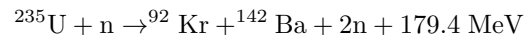


for neutrinos and antineutrinos, $m_\nu < 2.8 \text{ eV}/c^2$

7. Gamma decay occurs when a nucleus goes from a excited energy state and decays to a lower energy state.

$$E \sim 1 \text{ MeV} \rightarrow \lambda = hc/E = 1.24 \times 10^{-3} \text{ nm}$$

8. Fission was discovered by Lise Meitner with her nephew Otto Frisch and Otto Hahn who got his Nobel prize in chemistry in 1944.



9. Fission fragments smaller nuclei produced when a heavy atomic nucleus and chain reactions occur when these fission fragments again produce nuclear fission.

XIV. PARTICLE PHYSICS (4TH, JUNE)

1. Nuclear fission, fusion, and nuclear reactors (pressurized water reactors) might be on the test.

XV. PARTICLE PHYSICS (11TH, JUNE)

1. Particles can be categorised like the following

category	type
Fermions and Bosons	electrons, protons; photons and pions
Hadrons and Leptons	Baryons, Mesons; electrons and muons
Particles and Antiparticles	electrons; positrons

XVI. PARTICLE PHYSICS (13TH, JUNE)

- 1.

2. The lepton number is conserved through interactions, and therefore leptons are always produced when electrons are introduced.
- 3.

