The Hall Effect and Helmholtz Coils

Sejin Jeon
Sogang University Physics Department

(2nd Week Pre-Experiment Lab Report)

I. RESEARCH QUESTIONS

a. Understanding the Hall effect and ways of measuring it.

b. Measuring a sample's charge carriers' sign (positive or negative[1]) and charge density, and understanding n-type and p-type semiconductors in the process.

c. Understanding how one could utilise the Hall effect to measure (strength of) a field, and discussing the method's limitations.

d. Understanding the functions of Helmholtz coils, acknowledging real-world applications.

II. THEORY

A. The Hall Effect

1. Magnetic Fields and its Magnetic Flux

Magnetic fields [7] are vector fields that describe how magnetic forces are exerted in regions of space. Created by magnetic materials, magnetic fields are represented by imaginary lines called "field lines" whose role lie in indicating the direction of the magnetic force at a given point. The *Magnetic flux* [1] of a magnetic field, on the other hand, refers to a quantity that describes the amount of magnetic field passing through a specific surface. Mathematically, the quantity is described by the following equation.

$$\Phi = BA\cos\theta \tag{1}$$

The unit of magnetic flux is given in webers (Wb), which can be taken as the unit of magnetic field strength, the tesla (T), times the square meter (m^2).

2. Gauss's Law

Gauss's law states that the total electric flux through any closed surface is equivalent to the total charge enclosed by the closed surface divided by some constant (the permittivity constant of the medium that the electric flux permeates through). Formally, the law can be stated in two forms, the integral form and the differential form, as follows.

$$\oint_S \mathbf{E}\cdot d\mathbf{A} = \frac{q_{\text{enc}}}{\varepsilon_0} \tag{2}$$

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0} \tag{3}$$

Where (2) is the integral form and (3) is the differential form.

3. The Lorentz Force

The *Lorentz force* is a type of force felt by a charge particle that travels through space that is occupied by a magnetic field and/or electric field. The force accounts for both electric and magnetic influences, formally stated by the formula that follows.

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \tag{4}$$

The cross product implicitly shows the fact that the angle between the magnetic field and the velocity is relevant to the force that the charged particle experiences. Due to this law, it is expected that in the first experiment, as the current increases, the voltage difference across the sample will increase, as a higher electric force would be required to counter the magnetic force that is proportional to the current.

4. The Hall Effect

Discovered by the American physicist Edwin Hall, the *Hall effect* [6] describes a generation of a voltage difference across an electrical conductor that conducts an electrical current through charge carriers, when placed in a space occupied by a magnetic field. The *Hall voltage* [6], given by the following formula, gives the precise voltage between the ends of the conductor, when measured in a particular setup, identical to the setup used in this experiment.

$$\frac{\Delta V_H}{w} = v_d B = \frac{j}{ne}B = \frac{i}{wtne}B \tag{5}$$

The following equation can be solved for n, the density of the charge carriers, as the equation below.

$$n = \frac{iB}{et\Delta V_H} \tag{6}$$

[1] Personal notes and records are in red text.

The *Hall coefficient* is given as the following, which can also be taken as the proportionality constant between the electric field and the magnetic field times the charge density j.

$$R_H = \frac{1}{ne} = \frac{E}{jB} \tag{7}$$

Here, the electric field and the magnetic field are perpendicular from each other. In this experiment, knowing that the charge carriers are electrons, we can use the formula to easily calculate the Hall coefficient from the electron charge e and the charge carrier density calculated above.

5. Charge Density and Charge Carrier Density

Charge density refers to the amount of how packed charge is in a given volume, in par with the definition of other densities described in physics. Often denoted by the symbol ρ, it can be expressed formally as the electric charge per unit volume as following.

$$Q = \int_V \rho(\mathbf{r})\, dV \tag{8}$$

where the variables have their usual definitions. On the other hand, *charge carrier density* [5] is the number of charge carriers per unit volume, conventionally denoted by the number n. Also called carrier concentration, it is given by the following formula.

$$N = \int_V n(\mathbf{r})\, dV \tag{9}$$

Where N denotes the total number of charges in the given volume V.

6. Semiconductors

Semiconductors [3] are materials that have conductivity between insulators and conductors. They have useful applications in electronics with their various properties, including generally having resistivity falling in higher temperatures. At temperatures above absolute zero, the thermal energy within the semiconductor will cause a small fraction of the covalent electrons to break loose of their orbits and become *conduction electrons*. What is left behind of that breaking is what is called a *hole*, a void that is capable of accepting a new electron.

A high number of conduction electrons can be obtained through a process called *doping*, where one would introduce suitable impurity atoms to the system. The number of conducting electrons and holes determine the type of semiconductor. Semiconductors that contain few holes and relatively many conducting electrons are named *N-type semiconductors*, whereas in the opposite case (many holes and few mobile electrons), the semiconductor would be called a *P-type semiconductor*. The N and P refers to the charge of what the semiconductors have more, that is, the negative charge that the electrons carry and the positive charge that the holes carry.

7. Hall Probes and the Gauss Meter

Hall probes, also known as Hall effect sensors, have their role directly measuring the strength of the magnetic field. It utilises the hall effect, where it measures the voltage difference created within the device due to the surrounding magnetic field. The *Gauss Meter* is a term used to refer to instruments that measure magnetic field strength, irrelevant to the method used.

B. Helmholtz Coils

1. Ampère's Law and the Bio-Savart Law

Ampère's Law states that the current through a closed surface can be calculated through integrating the line integral of the magnetic field through the boundary of the closed surface or vice-versa. Alike Gauss' law, Ampère's law has two forms.

$$\oint_C B \cdot d\mathbf{l} = \mu_0 I_{\text{enc}} \tag{10}$$

$$\nabla \times B = \mu_0 \mathbf{J} \tag{11}$$

where equation (10) is the integral form and equation (11) is the differential form.

The *Bio-Savart Law* describes the force experienced by a charged particle moving in a magnetic field, where the charged particle is most often described as a current passing through a given a closed path. In this certain experiment, due to the inverse-square law of the following equation, it is predicted that the field across the two rings remain constant through different distance intervals.

$$\mathbf{B} = \frac{\mu_0}{4\pi} \int_C \frac{I\, d\mathbf{l} \times \mathbf{r}}{r^3} \tag{12}$$

2. The Applications of Ampère's Law

Ampère's law of calculating magnetic fields allows us to calculate the magnetic field of a solenoid, given an ideal case. When drawing a rectangular surface such that the Ampèrian surface only has to account for a length L that is parallel to the side of the field, the following derivation can be made to find the magnetic field within the solenoid.

$$BL = \mu_0 N i$$

$$B = \mu_0 \frac{Ni}{L} \tag{13}$$

$$B = \mu_0 n i$$

In an non-ideal case, the following derivation would only act as a close approximation.

3. The Helmholtz Coil

Helmholtz coils are devices that generate near-uniform magnetic fields. It consists of two circular magnetic coils, or electromagnets, that are placed symmetrically along a common axis, separated by some distance. Normally, the coils send currents in the same direction, whereas when the currents are sent in different directions, the coils would be called *anti-Helmholtz coils*. They are often used to take away the earth's magnetic field influence on a certain apparatus that requires minimal field strength that would affect it.

III. APPARATUS AND SAMPLES

A. The Hall Effect Experiment Set

FIG. 1: A digital Gauss meter and a constant power supply (for the electromagnet [4]).

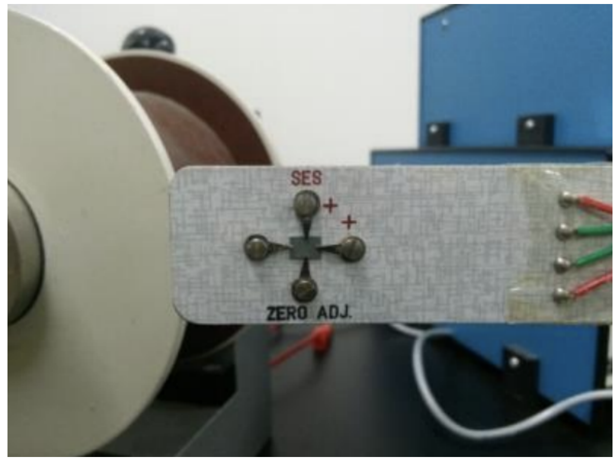

FIG. 2: A Hall probe (Ge crystal).

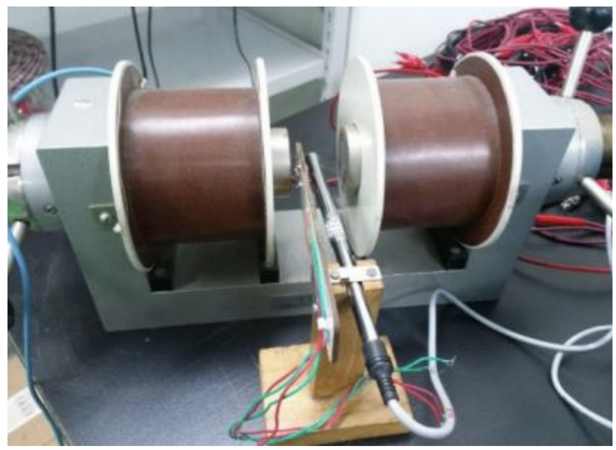

FIG. 3: A Gauss meter planted on the electromagnet.

FIG. 4: The Hall effect experiment setup consisting of a constant current provider and a digital milivoltmeter.

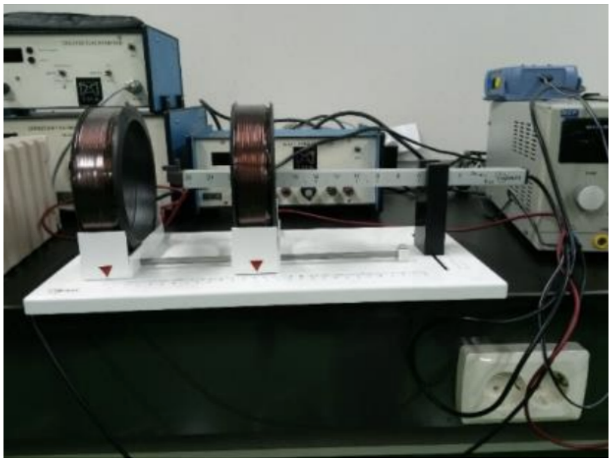

FIG. 5: A Helmholtz coil with a Gauss meter.

FIG. 6: A power supply.

B. The Helmholtz Coil Experiment Set

IV. METHODOLOGY

A. The Hall Effect

1. Set up the Hall effect experiment set. Place the sample and the Gauss meter between the electromagnets and connect the electromagnet and the sample to the constant current power supply and the Hall effect setup respectively.

2. By doing this, one sends a current on one side of the sample and measures the voltage on the other. The method of connecting the sample and the hall effect set up is as shown in the figure below. (The reason to why this connecting method is valid is entirely due to the Bio-Savart Law, where electrons are pushed in a direction perpendicular both to the electric field (which is outwards in the figure) and the direction of the current (which is towards the $+x$-direction in reference to the page). Note that this geometrical characteristic of the Lorentz force is the whole basis behind the theory of the Hall effect.)

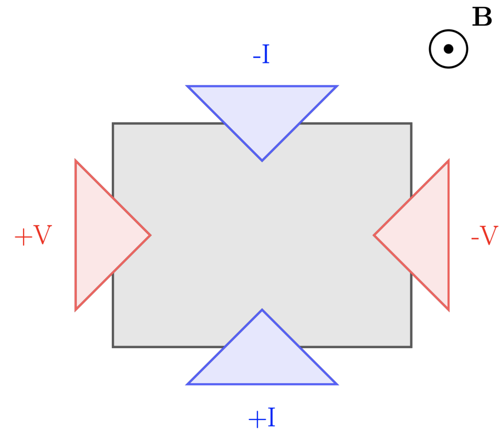

FIG. 7: The Hall effect experiment setup.

3. Here, the sample's thickness is 0.5 mm, and the horizontal and vertical widths need to be measured as they vary by sample.

4. When the equipment is all set, first only place the Gauss meter between the electromagnets and measure the magnetic field between the them.

5. Vary the current by 6 different levels and measure the magnetic field strength for each. By doing this, find the relationship between the current and the magnetic field strength. In the process, the upper limit of the value of the current is 1.5 A.

6. The magnetic field strength can be varied not only by changing the current but also by changing the distance between the metal rods that are attached to the electromagnets. (The field is expected to increase in intensity as the distance decreases.)

7. Again place the sample between the electromagnets. Measure the voltage when a current is sent through the sample and when the magnetic field is set to zero. In the process, vary the current by 5 different levels, recording the voltage for each. Through this process, choose an adequate value of the current for a Hall measurement. An adequate value can be more precisely described as a value within a range of values that allow the current-voltage relationship to show a linear regression.

8. Set the current to be a apt value, and measure the magnetic field's strength as it varies by 10 different values.

9. Change the sample and repeat steps 4 through 5. Find the Hall coefficient and the density of the charge carriers through the data collected. This experiment allows one to specify the type and the density of the charge carrier.

10. Investigate methods that could be used to measure a magnetic fields strength without a Gauss meter

through using the samples that were used in the experiment. Additionally, compare the investigated method and the case where one would use an actual Gauss meter. Remove the Gauss meter and set an arbitrary magnetic field, measuring the field using the samples. Next, use the Gauss meter to measure the arbitrary field and repeat the field measurement three times. When using the samples for measurement, use the recorded information from the experiments done beforehand.

B. The Helmholtz Coil

1. Set up the Helmholtz coil experiment. In the process, connect the coil and the power supply such that the current flowing through the two coils have the same direction (moreover, the Gauss meter's zero point must be calibrated). This is due to the fact that because the Gauss meter, unlike the probe used in the previous Hall effect experiment, can measure the magnetic fields of both axes, resulting in an incorrect measurement when not verified.
2. Measure the radius of the coil and through the measured radius, calculate the distance between the two coils that ensures a uniform intensity of the magnetic field between them.
3. Based on the distance calculated beforehand between the two coils, measure the magnetic field strength between the coils when the distance is taken to be equal to, smaller than, and larger than that initial distance. The distance between the coils will be varied in 5 ways, and when measuring the magnetic field intensity between the coils, measurements must be taken per 1 cm interval in reference to the center of the two coils (for example, if the distance between the coils is 8 cm, the center of the two coils will be set as 0 cm point, and the magnetic field strength measurements will be taken at points corresponding to the -4, -3, -2, -1, 0, 1, 2, 3, and 4 cm mark).
4. Repeat the experiment by varying the intensity of the current flowing through the coils, selecting only cases where the magnetic field between the two coils remain constant.
5. Discuss whether there would be a method of neutralizing the Earth's magnetic field using a Helmholtz coil.

[1] Griffiths, D. Introduction to electrodynamics. *Pearson Education,* (2014), pp. 66–70, 210–212.
[2] Halliday, D., Resnick, R., and Krane, K. Physics, volume 2. *Wiley, 2nd Edition* (2010), pp. 734–736.
[3] Hu, C. Modern semiconductor devices for integrated circuits. *Prentice Hall,* (2010), pp. 1–6.
[4] The Sogang University Physics Department. Experimental physics 1 manual. *"Hall Effect, Helmholtz Coil"*.
[5] Wikipedia. Charge carrier density. `https://en.wikipedia.org/wiki/Charge_carrier_density`.
[6] Wikipedia. Hall effect. `https://en.wikipedia.org/wiki/Hall_effect`.
[7] Wikipedia. Magnetic field. `https://en.wikipedia.org/wiki/Magnetic_field`.