Electronic Measurement Instruments & Elementary Circuit Theory 2

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(4nd Week Post-Experiment Lab Report)

I. EXPERIMENT DATA & ANALYSIS

A. Thévenin's Theorem

Thevenin resistance		Thevenin voltage	
experimental (kΩ)	theoretical (kΩ)	experimental (V)	theoretical (V)
1.049	1.010	2.483	2.500

TABLE I: The resistance between A and B without external power (R_{TH}) and the voltage between A and B when 10 V is applied (V_{TH}).

In the table above, the figure that should be referenced is FIG 1-1. In calculating the theoretical Thevenin resistance, basic circuit theory was used. Assuming that the power source is a closed circuit, the equivalent resistance between points A and B can be calculated as the following.

$$R_3 + \frac{R_1 R_2}{R_1 + R_2} = 0.510\ \mathrm{k\Omega} + \frac{1.000\ \mathrm{k\Omega} \times 1.000\ \mathrm{k\Omega}}{1\ \mathrm{k\Omega} + 1\ \mathrm{k\Omega}}$$
$$= 1.010\ \mathrm{k\Omega}$$

On the other than, the theoretical Thevenin voltage was found through the resistance formula for series resistors. The fact that the voltage across a certain resistor is proportional to the resistance of that resistor relative to the resistance of the full circuit was used.

$$\frac{R_2}{R_1 + R_2} V = \frac{1\ \mathrm{k\Omega}}{1\ \mathrm{k\Omega} + 1\ \mathrm{k\Omega}} \times 5\ \mathrm{V} = 2.5\ \mathrm{V}$$

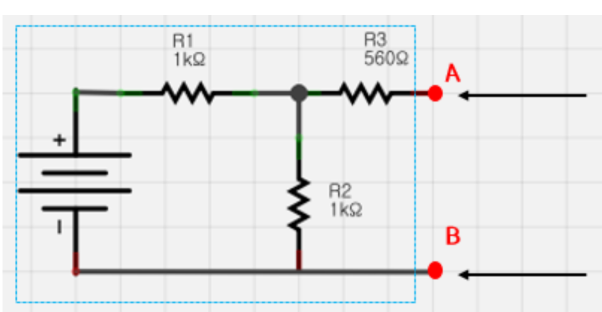

FIG 1-1. Thevenin's Theorem.

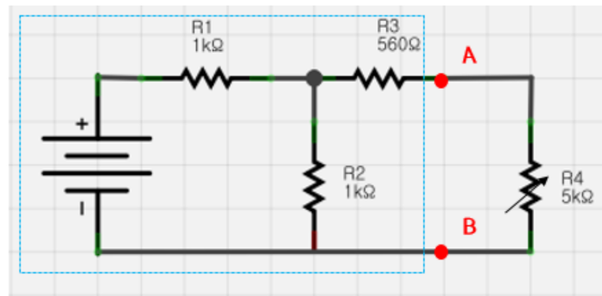

FIG 1-2. Thevenin's Theorem.

The percentage differences between the theoretical and experimental resistances and voltages can be found to be 0.2640 % and 0.3564 % respectively, showing extremely low error levels. The reasons for these levels will be further explored in the next part of this lab report. The formula used in investigating these differences can be written as the following.

$$V\ \%\ \text{error} = \frac{|V_{\text{theoretical}} - V_{\text{actual}}|}{V_{\text{theoretical}}} \tag{1}$$

The same formula was used for resistances, where V_i was replaced with R_i.

R_4 (kΩ)	V experimental (V)	V theoretical (V)	% diff.
0.523	0.865	0.853	1.41
1.126	1.158	1.318	12.14
2.297	1.974	1.736	13.71

TABLE II: The voltage across R_4 with 3 different arbitrary values.

In the table above, the voltage (V) across the the fourth resistor in FIG 1-2. was obtained through a multimeter. The theoretical voltage was obtained using the theoretical Thevenin voltage and the theoretical Thevenin resistance, along with the variable resistor measurement made using the multimeter.

$$V_{R_4} = \frac{R_4}{R_{\mathrm{Th}} + R_4} V_{\mathrm{Th}} \tag{2}$$

Where R_{Th} and V_{Th} are the Thevenin resistance and voltage respectively.

R_4 (Ω)	Voltage (V) experimental	Voltage (V) theoretical	Current (mA) experimental	Current (mA) theoretical	Power (mW) experimental	Power (mW) theoretical
100	1.62	1.32	13.5	13.2	21.9	17.4
200	1.88	2.08	10.2	10.4	19.2	21.6
300	2.69	2.59	8.7	8.6	23.4	22.3
400	3.34	2.94	7.8	7.4	26.1	21.8
500	2.71	3.21	6.1	6.4	16.5	20.5

TABLE III: Change of voltage, current, and power across the variable resistor when the resistance is varied (R_1 = 560 Ω, R_2 = 560 Ω, and V = 5 V).

R_4 (Ω)	Voltage (V) experimental	Voltage (V) theoretical	Current (mA) experimental	Current (mA) theoretical	Power (mW) experimental	Power (mW) theoretical
100	1.49	1.09	13.4	13.2	20	17.4
200	1.49	1.79	10.0	10.4	14.9	21.6
300	2.48	2.28	8.9	8.6	22.1	22.3
400	2.54	2.64	7.3	7.4	18.5	21.8
500	3.41	2.91	6.9	6.4	23.5	20.5

TABLE IV: Change of voltage, current, and power across the variable resistor when the resistance is varied (R_1 = 1000 Ω, R_2 = 560 Ω, and V = 5 V).

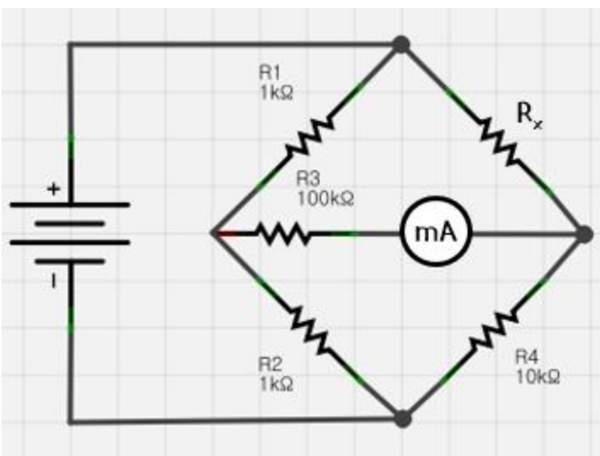

FIG 2. Wheatstone Bridges and Maximum Power Transmission.

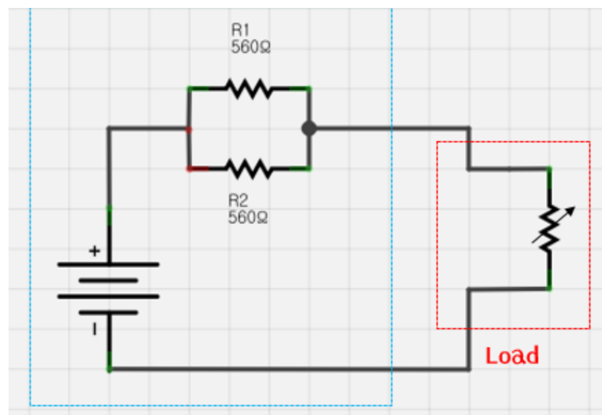

FIG 3. Wheatstone Bridges and Maximum Power Transmission.

The last measurement set in the first experiment was in finding the resistance of the variable resistor R_4 such that the voltage sent by the power source was halved to be sent to R_4.

R_4	experimental R (kΩ)	theoretical R (kΩ)
1.250	1.055	1.021

TABLE V: Resistance of the variable resistor R_4 such that the voltage sent by the power source was halved to be sent to R_4.

The percentage difference between the two values came out to be extremely low, being 3.333 %. The reason for this error will be further investigated in the second part of this lab report. The theoretical value for this measurement was again found from the voltage formula denoted above, equation (1). The significance of this voltage value can be identified through the equation, where the resistance value that we obtained is the resistance value of R_4 where the coefficient of the Thevenin voltage is 1/2. In other words, the resistance we found of R_4 that satisfies the proposed condition is the *exact Thevenin resistance of the arbitrary circuit.* Further implications will again be made in the next section.

B. Wheatstone Bridges and Maximum Power Transmission

R_X (Ω)	R_4 experimental (Ω)	R_4 theoretical (Ω)
560	611	560
470	503	470
335	387	335

TABLE VI: Resistances of R_4 for 0 current for three different arbitrary values of R_X (R_3 not removed).

The theoretical values were the actual values of R_X, given the theory of Wheatstone bridges. The percentage differences for each column are 9.11 %, 7.02 %, and 9.01 % for the first, second, and three rows respectively.

R_X (Ω)	R_4 experimental (Ω)	R_4 theoretical (Ω)
560	585	560
470	477	470
335	318	335

TABLE VII: Resistances of R_4 for 0 current for three different arbitrary values of R_X (R_3 removed).

The second measurement for the second experiment was done in conjunction with the first measurement, where measurements were made with and without the R_3 resistor. The percentage differences for this measurement for each column are 4.46 %, 0.85 %, and 0.05 % for the first, second, and three rows respectively. The substantial decrease in error implies higher accuracy when the R_3 was removed, showing that the choice of whether to use or not use R_3 should depend on whether dealing with high ranges of current across the multimeter or low ranges. Further discussions are to be made in the second section of this report.

The last two measurements made, seen in tables VII and III, were on finding the variation of power dissipated on the load depending on resistance of the load. The graphing of the data can be seen in the *Python* plot below, created using the *Matplotlib library* for mathematical plotting.

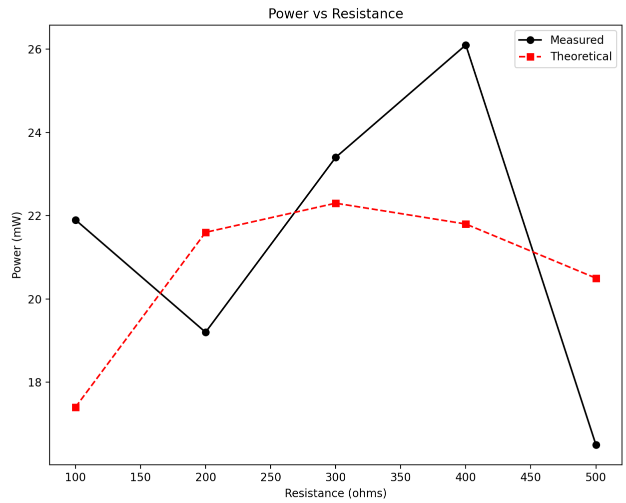

FIG. 1: Change of power across the variable resistor when the resistance is varied (R_1 = 560 Ω, R_2 = 560 Ω, and V = 5 V).

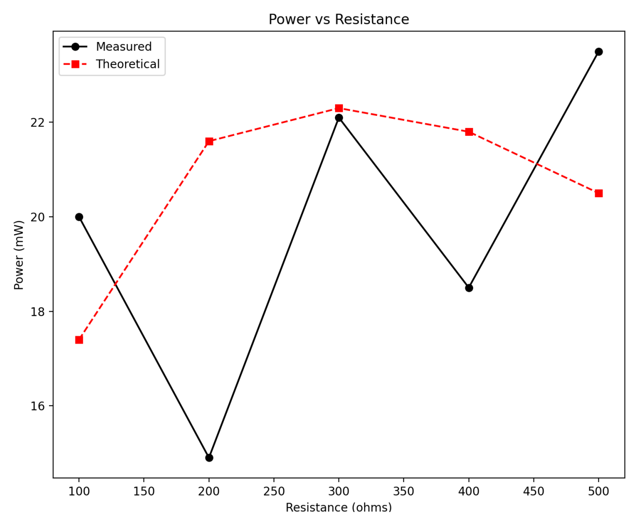

FIG. 2: Change power across the variable resistor when the resistance is varied (R_1 = 1000 Ω, R_2 = 560 Ω, and V = 5 V).

II. DISCUSSION

A. Goals and Recapitulation of Experiments

For the whole experiment, there were two sub-experiments, aimed at obtaining a total of two goals. The two goals were:

a. Understanding the principles of electronic measuring instruments and basic equipment and being able to use them.

b. Conducting basic experiments on circuits using the *NI ELVIS* circuit board and thus understanding the usage of it. Additionally, understanding the fundamental concepts behind elementary circuit theory.

Consequently, in total, there were 4 different sets of measurements made for the first experiment, 4 different sets of measurements made for the second experiment. The 8 experiments can be seen in the list below.

1. The first measurement (set) of the first experiment was on measuring the Thevenin resistance for FIG 1-1. The

results can be seen in I.

2. The second measurement (set) of the first experiment was on measuring the Thevenin voltage for FIG 1-1. The results can be seen in I.

3. The third measurement (set) of the first experiment was on measuring the voltage across R_4 for different variable resistance values in FIG 1-2. The results can be seen in II.

4. The third measurement (set) of the first experiment was on finding the voltage across R_4 that allowed exactly half of voltage of the power source to be distributed to it. Like the measurement (set) made beforehand, the circuit in FIG 1-2. was used. The results can be seen in V.

5. The first measurement (set) of the second experiment was on measuring the variable resistance R_X indirectly using the Wheatstone bridge of FIG 3. For higher range but lower resolution, R_3 was set as 1000 kΩ. The results can be seen in VI.

6. The second measurement (set) of the second experiment was on measuring the variable resistance R_X indirectly using the Wheatstone bridge of FIG 3. For lower range but higher resolution, R_3 was set as 0 kΩ. The results can be seen in VII.

7. The third measurement (set) of the second experiment was on measuring the power dissipated but the load (variable resistor) when the resistance was varied from 100 Ωto 500 Ω. R_1 was set as 560 Ω, R_2 was set as 560 Ω, and the voltage was 5 V. The results can be seen in III.

8. The fourth measurement (set) of the second experiment was on measuring the power dissipated but the load (variable resistor) when the resistance was varied from 100 Ωto 500 Ω. R_1 was set as 1000 Ω, R_2 was set as 560 Ω, and the voltage was 5 V. The results can be seen in IV.

B. Evaluation and Error Assessment

In the first and second measurements of the first experiment, sound results were made, with the theoretical and experimental values of the Thevenin voltages and resistances well aligning with each other. The percentage errors were also extremely low as shown above, thus implying a less urgency of evaluating error. The second measurement, where the voltage across the variable resistor was measured, the percentage difference ranged from 1 % to 14 %, showing some error in the measurement. A likely source of this error is faulty connection between the terminals.

Faulty connection between terminals lead to high resistance at the connection point. This resistance can create a voltage drop across the circuit, resulting in an inaccurate reading. Additionally, environmental factors such as electromagnetic interference or fluctuations in power supply can introduce noise into the measurement, distorting the true voltage value. Ensuring proper calibration of measurement equipment and minimizing external interference are crucial steps to mitigating such errors and obtaining reliable voltage readings.

In The last measurement set of the first experiment set, where the resistance of the variable resistor $R4$ such that the voltage sent by the power source was halved to be sent to $R4$ was sought, very little error occurred, requiring no more extensive analysis.

In the first and second measurement of the second experiment, the Wheatstone bridge proved to be a very accurate way of measuring resistance, albeit less accurate relative to the millimeter. The error decreased from a range around 10 % to a range around 1 % when R_3 was eliminated, showing the less resistance with the ammeter indeed results in a more accurate measurement.

In the third and fourth measurements of the second experiment, the plots made by *Python* trends being rather disrupted and random. The main goal of this experiment was to find whether the maximum power indeed occurs when the resistance of the load is identical with the Thevenin resistance of the system. The derivation is rather trivial, seen as the following.

$$P_{\text{load}} = \frac{V_{\text{Th}}^2}{(R_{\text{Th}} + R_{\text{load}})^2} \times R_{\text{load}}$$

With the power loss of the load expressed in terms of the Thevenin quantities and the load's resistance as shown above, the equation can be partially derivated to result in the following.

$$\frac{\partial P}{\partial R_{\text{load}}} = \frac{2V_{\text{Th}}^2(R_{\text{Th}} + R_{\text{load}})}{(R_{\text{Th}} + R_{\text{load}})^4} - \frac{2V_{\text{Th}}^2 R_{\text{load}}}{(R_{\text{Th}} + R_{\text{load}})^3}$$

The equation equal zero for different values of R_{load}, but the only positive value is when R_{load} is equal to R_{Th}, implying that this is when the power reaches its maximum value. Thus, the graphs and their results are well justified, where the maximum values are well shown. However, local fluctuations cannot be ignored, with the value of the power at 500 Ωseeming to have a jump in the second graph and etcetera. These errors can be attributed to a number of causes such as the following.

Improper circuit wiring presents a significant potential source of disruption, possibly causing erratic signal behavior and improperly fluctuating currents. Both voltage and current measurements in the third and fourth experiments displayed substantial variability over time, characterized by frequent fluctuations. Addressing such errors would involve utilising higher voltages and currents and thus decreasing the impact of small fluctuations in the results.

Voltage drops are a common occurrence in electronic systems, and can disrupt experimental setups. It poses a significant threat to this study. It's likely that cable resistance and grounding issues during experimentation caused sudden and frequent voltage fluctuations, impacting the accuracy of measurements. To mitigate this, employing higher voltages, as previously mentioned, is recommended as a potential solution.

[1] Halliday, D., Resnick, R., and Krane, K. Physics, volume 2. *Wiley, 2nd Edition* (2010), pp. 845–852.

[2] The Sogang University Physics Department. Experimental physics 1 manual. *"Electronic Measurement Instruments, Elementary Circuit Theory 2"*.

[3] Wikipedia. Maximum power transfer theorem. https://en.wikipedia.org/wiki/Maximum_power_transfer_theorem.

[4] Wikipedia. Thévenin's theorem. https://en.wikipedia.org/wiki/Th%C3%A9venin%27s_theorem.

[5] Wikipedia. Wheatstone bridge. https://en.wikipedia.org/wiki/Wheatstone_bridge.